Exercise 2B

1 a $x^2 + 3x + 1 = 0$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2 \times 1}$$
$$= \frac{-3 \pm \sqrt{9-4}}{2}$$
$$= \frac{-3 \pm \sqrt{5}}{2}$$

So $x = \frac{-3+\sqrt{5}}{2}$ or $x = \frac{-3-\sqrt{5}}{2}$

b $x^2 - 3x - 2 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2 \times 1}$$
$$= \frac{3 \pm \sqrt{9+8}}{2}$$
$$= \frac{3 \pm \sqrt{17}}{2}$$

So $x = \frac{3+\sqrt{17}}{2}$ or $x = \frac{3-\sqrt{17}}{2}$

c $x^2 + 6x + 6 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2 \times 1}$$
$$= \frac{-6 \pm \sqrt{36-24}}{2}$$
$$= \frac{-6 \pm \sqrt{12}}{2}$$
$$= \frac{-6 \pm \sqrt{4 \times 3}}{2}$$
$$= \frac{-6 \pm 2\sqrt{3}}{2}$$
$$= -3 \pm \sqrt{3}$$

So $x = -3+\sqrt{3}$ or $x = -3-\sqrt{3}$

d $x^2 - 5x - 2 = 0$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-2)}}{2 \times 1}$$
$$= \frac{5 \pm \sqrt{25+8}}{2}$$
$$= \frac{5 \pm \sqrt{33}}{2}$$

So $x = \frac{5+\sqrt{33}}{2}$ or $x = \frac{5-\sqrt{33}}{2}$

e $3x^2 + 10x - 2 = 0$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(3)(-2)}}{2 \times 3}$$
$$= \frac{-10 \pm \sqrt{100+24}}{6}$$
$$= \frac{-10 \pm \sqrt{124}}{6}$$
$$= \frac{-10 \pm \sqrt{4 \times 31}}{6}$$
$$= \frac{-10 \pm 2\sqrt{31}}{6}$$
$$= \frac{-5 \pm \sqrt{31}}{3}$$

So, $x = \frac{-5+\sqrt{31}}{3}$ or $x = \frac{-5-\sqrt{31}}{3}$

f $4x^2 - 4x - 1 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-1)}}{2 \times 4}$$
$$= \frac{4 \pm \sqrt{16+16}}{8}$$
$$= \frac{4 \pm \sqrt{32}}{8}$$
$$= \frac{4 \pm \sqrt{16 \times 2}}{8}$$
$$= \frac{4 \pm 4\sqrt{2}}{8}$$
$$= \frac{1 \pm \sqrt{2}}{2}$$

So $x = \frac{1+\sqrt{2}}{2}$ or $x = \frac{1-\sqrt{2}}{2}$

g $4x^2 - 7x = 2$

$4x^2 - 7x - 2 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(4)(-2)}}{2 \times 4}$$

$$= \frac{7 \pm \sqrt{49 + 32}}{8}$$

$$= \frac{7 \pm \sqrt{81}}{8}$$

$$= \frac{7 \pm 9}{8}$$

So $x = 2$ or $x = -\frac{1}{4}$

h $11x^2 + 2x - 7 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(11)(-7)}}{2 \times 11}$$

$$= \frac{-2 \pm \sqrt{4 + 308}}{22}$$

$$= \frac{-2 \pm \sqrt{312}}{22}$$

$$= \frac{-2 \pm \sqrt{4 \times 78}}{22}$$

$$= \frac{-2 \pm 2\sqrt{78}}{22}$$

$$= \frac{-1 \pm \sqrt{78}}{11}$$

So $x = \frac{-1 + \sqrt{78}}{11}$ or $x = \frac{-1 - \sqrt{78}}{11}$

2 a $x^2 + 4x + 2 = 0$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2 \times 1}$$

$$= \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{-4 \pm \sqrt{8}}{2}$$

So $x = -0.586$ or $x = -3.41$

b $x^2 - 8x + 1 = 0$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(1)}}{2 \times 1}$$

$$= \frac{8 \pm \sqrt{64 - 4}}{2}$$

$$= \frac{8 \pm \sqrt{60}}{2}$$

So $x = 7.87$ or $x = 0.127$

c $x^2 + 11x - 9 = 0$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(1)(-9)}}{2 \times 1}$$

$$= \frac{-11 \pm \sqrt{121 + 36}}{2}$$

$$= \frac{-11 \pm \sqrt{157}}{2}$$

So $x = 0.765$ or $x = -11.8$

d $x^2 - 7x - 17 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-17)}}{2 \times 1}$$

$$= \frac{7 \pm \sqrt{49 + 68}}{2}$$

$$= \frac{7 \pm \sqrt{117}}{2}$$

So $x = 8.91$ or $x = -1.91$

e $5x^2 + 9x - 1 = 0$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(5)(-1)}}{2 \times 5}$$

$$= \frac{-9 \pm \sqrt{81 + 20}}{10}$$

$$= \frac{-9 \pm \sqrt{101}}{10}$$

So $x = 0.105$ or $x = -1.90$

f $2x^2 - 3x - 18 = 0$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-18)}}{2 \times 2}$$

$$= \frac{3 \pm \sqrt{9 + 144}}{4}$$

$$= \frac{3 \pm \sqrt{153}}{4}$$

So $x = 3.84$ or $x = -2.34$

2 g $3x^2 + 8 = 16x$

$3x^2 - 16x + 8 = 0$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(3)(8)}}{2 \times 3}$$

$$= \frac{16 \pm \sqrt{256 - 96}}{6}$$

$$= \frac{16 \pm \sqrt{160}}{6}$$

So $x = 4.77$ or $x = 0.558$

h $2x^2 + 11x = 5x^2 - 18$

$3x^2 - 11x - 18 = 0$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(-18)}}{2 \times 3}$$

$$= \frac{11 \pm \sqrt{121 + 216}}{6}$$

$$= \frac{11 \pm \sqrt{337}}{6}$$

So $x = 4.89$ or $x = -1.23$

3 a $x^2 + 8x + 12 = 0$

$(x + 6)(x + 2) = 0$

$x + 6 = 0$ or $x + 2 = 0$

So $x = -6$ or $x = -2$

b $x^2 + 9x - 11 = 0$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(1)(-11)}}{2 \times 1}$$

$$= \frac{-9 \pm \sqrt{81 + 44}}{2}$$

$$= \frac{-9 \pm \sqrt{125}}{2}$$

So $x = 1.09$ or $x = -10.1$

c $x^2 - 9x - 1 = 0$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(1)(-1)}}{2 \times 1}$$

$$= \frac{9 \pm \sqrt{81 + 4}}{2}$$

$$= \frac{9 \pm \sqrt{85}}{2}$$

So $x = 9.11$ or $x = -0.110$

d $2x^2 + 5x + 2 = 0$

$(2x + 1)(x + 2) = 0$

$2x + 1 = 0$ or $x + 2 = 0$

So $x = -\frac{1}{2}$ or $x = -2$

e $(2x + 8)^2 = 100$

$2x + 8 = \pm 10$

$x + 4 = \pm 5$

$x = -4 \pm 5$

So $x = 1$ or $x = -9$

f $6x^2 + 6 = 12x$

$6x^2 - 12x + 6 = 0$

$6(x^2 - 2x + 1) = 0$

$6(x - 1)(x - 1) = 0$

$x - 1 = 0$

So $x = 1$

g $2x^2 - 11 = 7x$

$2x^2 - 7x - 11 = 0$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-11)}}{2 \times 2}$$

$$= \frac{7 \pm \sqrt{49 + 88}}{4}$$

$$= \frac{7 \pm \sqrt{137}}{4}$$

So $x = 4.68$ or $x = -1.18$

h $x = \sqrt{8x - 15}$

$x^2 = 8x - 15$

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x - 3 = 0$ or $x - 5 = 0$

So $x = 3$ or $x = 5$

4 Area of trapezium $= 50$

$\frac{1}{2}(2x)(x + (x + 10)) = 50$

$x(2x + 10) = 50$

$x^2 + 5x - 25 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-25)}}{2 \times 1}$$

$$= \frac{-5 \pm \sqrt{25 + 100}}{2}$$

$$= \frac{-5 \pm \sqrt{125}}{2}$$

$$= \frac{-5 \pm \sqrt{25 \times 5}}{2}$$

$$= \frac{-5 \pm 5\sqrt{5}}{2}$$

Height $= 2x = -5 \pm 5\sqrt{5} = 5\left(\pm\sqrt{5} - 1\right)$

Height cannot be negative, so height is $5\left(\sqrt{5} - 1\right)$ m.

Challenge

$$\frac{1}{x}+\frac{1}{x+2}=\frac{28}{195}$$

$$\frac{195}{x}+\frac{195}{x+2}=28$$

$$195+\frac{195x}{x+2}=28x$$

$$195(x+2)+195x=28x(x+2)$$

$$28x^2-334x-390=0$$

$$28x^2-334x-390=0$$

$$x=\frac{-(-334)\pm\sqrt{(-334)^2-4(28)(-390)}}{2\times 28}$$

$$=\frac{334\pm\sqrt{111\,556+43\,680}}{56}$$

$$=\frac{334\pm\sqrt{155\,236}}{56}$$

$$x=13 \text{ or } x=-\frac{15}{14}$$

It is given that x is positive,
so the solution $x = 13$.